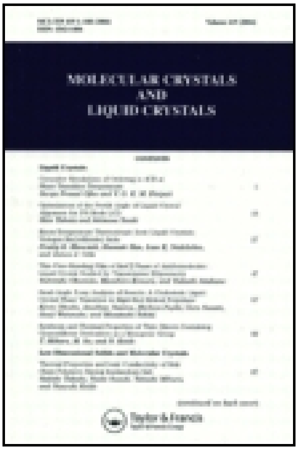

Molecular Crystals and Liquid Crystals

Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/gmcl16

Fractional Statistics and Fractional Charge in Low Dimensional Metals

J. R. Schrieffer [a]

[a] Institute for Theoretical Physics, University of California, Santa Barbara, California, 93106

Version of record first published: 17 Oct 2011.

To cite this article: J. R. Schrieffer (1985): Fractional Statistics and Fractional Charge in Low Dimensional Metals, Molecular Crystals and Liquid Crystals, 118:1, 57-64

To link to this article: http://dx.doi.org/10.1080/00268948508076189

Mol. Cryst. Liq. Cryst. 1985, Vol. 118, pp. 57-64
0026-8941/85/1184-0057/$15.00/0

FRACTIONAL STATISTICS AND FRACTIONAL CHARGE IN LOW DIMENSIONAL METALS

J.R. SCHRIEFFER
Institute for Theoretical Physics, University of California
Santa Barbara, California 93106

Abstract Solitons in quasi one-dimensional commensurate charged density wave systems are known to carry peculiar quantum numbers such as fractional charge. These systems in principal exhibit long range order corresponding to a definite bonding pattern as one moves along a chain. Recently, it has been proposed that quasi particles of fractional charge are responsible for the fractional quantum Hall effect. In this case, however, the system is believed to be translationally invariant in the absence of excitations of that the analysis appropriate to the Peierls charge density wave does not hold in this case. The relationship between these two different contexts in which fractionally charged excitations have been proposed is discussed and the underlining unity of the phenomena is explained. Furthermore, the quantum statistics of fractionally charged excitations is derived and it is shown that instead of conventional Fermi or Bose statistics, there may be a mixed or fractional statistics which alters the behavior of the system.

INTRODUCTION

In quasi one-dimensional conductors[1] and in the quantum Hall effect[2] it has been proposed that there exists stable excitations of sharp fractional charge. In the former, if the Hamiltonian is invariant under the translation group of the distorted lattice and the conduction band has a filling factor $\nu = \frac{1}{m}$, the ground state is m-fold degenerate corresponding to discrete values of the charge density wave phase ϕ . Soliton excitations act as domain walls separating regions having different ground state phase and act as particles of well defined mass (M), charge (q), spin (s), etc. For example, for $\nu = \frac{1}{3}$, $q/e = \pm\frac{1}{3}, \pm\frac{2}{3}$. For $\nu = \frac{1}{2}$, as in prestine transpolyacetylene, while $q/e = \pm\frac{1}{2}$ per spin, the fractional charge is masked by the equal population of up and down spins leading to the observed pecular charge-spin relations, $q = 0$, $s = \frac{1}{2}$ or $q/e = \pm 1$, $s = 0$. To date, it has not been possible to directly observe fractional q/e in quasi one-dimensional conductors.

In the quantum Hall effect, the Hall conductivity σ_{xy} is observed to have plateaus when the fillings factor of the lowest Landau level is in the vicinity of a rational value $\nu_i = 1/m_i$, where m_i is an odd integer. Laughlin[2] proposed that these fractional plateaus are the consequence of the fractional charge $q/e = 1/m_i$ of quasi-particle excitations above a condensate gap Δ . Haldane[3] as well as Halperin[4] proposed that secondary plateaus of σ_{xy} observed near $\nu_i = n_i/m_i$ for integer n_i and odd m_i are due to condensates of fractionally charged quasi particles at a given level producing excitations at the next level of a given hierarchy having charge $\pm n_i/m_i$. Below we discuss the relation between the excitations in these systems as well as the statistics of the quasi particles.

QUASI 1D CONDUCTORS

In commensurate quasi 1D conductors the natural ratio of the period λ of the CDW and the lattice spacing a is a rational fraction $\lambda/a = n/m$. The commensurability energy selects out a discrete set of degenerate broken symmetry states with the CDW phase ϕ being locally pinned to a value which for $\lambda/a = \frac{1}{3}$ is $\phi = 0, \pm\frac{2\pi}{3}, \pm\frac{4\pi}{3}, \ldots$ If one makes a domain boundary between a region A with $\phi = 0$ to the left of x_0 and a region B with $\phi = \frac{2\pi}{3}$ to the right of x_0, in effect the entire CDW in region B is rigidly translated a distance $d = \lambda/3$, since a sequence of three such translations would bring the CDW back to its original form with $\phi = 2\pi$. The charge of the wall or soliton[5] is given by noting that a translation by $\lambda/3$ will remove a charge $\rho_0 \cdot \lambda/3$ from the vicinity of x_0, where ρ_0 is the CDW charge density per λ. Since the Peierls distortion occurs at $2k_F = 2\pi/\lambda$ there is one electron per spin per wavelength λ. Therefore the charge localized on the wall is $q/e = \pm\frac{2}{3}$ depending on whether the CDW in region B is displaced to the right (+) or left (-), the factor of 2 arising from spin. The other charge states $q/e = \mp \frac{1}{3}$ come from adding or subtracting an electron in a state localized at the wall in Peierls gap.

Thus, fractional charge in commensurate 1D conductors is a consequence of the CDW asymptotically apporaching one of its degenerate ground states as one moves a distance large compared to ξ from the soliton, where ξ is the soliton half width and $\xi \approx (W/2\Delta)a$. Here W is the conduction band width and 2Δ is the Peierls gap. The apparent rigidity of the CDW under displacement is the consequence of the stability of the commensurate electron phonon system in selecting out a discrete set of degenerate ground states. If the CDW did not asymptotically settle down into one of these ground states, q/e would not be fractional. Thus, energetics enforce the fractional quantization of charge in quasi one-dimensional conductors.

QUANTUM HALL EFFECT

While plateaus of the Hall conductivity σ_{xy} at integer filling factors ν can be generally understood in terms of a one electron picture, the fractional plateaus arise from electron-electron correlation effects. If the Landau level spacing $\hbar\omega_0 = e\hbar B_0/m$ is sufficiently large, one can neglect all but the lowest Landau level in the system dynamics for $\nu < 1$ at low temperature. In this case the Hamiltonian is simply the Coulomb interaction between electrons in the lowest Laudau level basis. While a natural first approximation for treating e-e correlations is in terms of a CDW, there exists no experimental evidence favoring such a condensate with long range static phase coherence. Were such a state to exist, one would expect nonlinear transport at low Hall field as in sliding CDW systems. Rather, it appears that while strong repulsive correlations exist, quantum fluctuations "melt" the classical 2D Wigner lattice, i.e. destroy long range ϕ coherence leading to a fluid rather than solid-like state.

Laughlin[2] proposed a variational ground state wavefunciton for $\nu = \frac{1}{m}$ building in repulsive pair correlations in a manner analogous to the familiar Jastrow-type function. Working in the symmetric gauge $\vec{A}(r) = \frac{1}{2}\vec{B}_0 \times \vec{r}$, Laughlin proposed the ground state with odd m

$$\psi_m = \prod_{j<k} (z_j - z_k)^m \exp(-\frac{1}{4}\sum_\ell |z_\ell|^2), \tag{1}$$

where $z_j = x_j + iy_j$, with B_0 perpendicular to the x,y plane. A quasi-hole excitation localized around z_0 is assumed to be described by

$$\psi_m^{+z_0} = \prod_i (z_i - z_0)\psi_m \tag{2}$$

while a quasi particle is described by

$$\psi_m^{-z_0} = \prod_i \left(\frac{\partial}{\partial z_i} - \frac{z_0}{a_0^2}\right)\psi_m \tag{3}$$

where $2\pi a_0^2 B_0 = \Phi_0 = hc/e$ is the flux quantum.

The charge of a quasi particle can be determined in several ways. Laughlin represented $|\psi_m^{+z_0}|^2$ as the exponential of an effective free energy, which turns out to be energy of a 2D one component plasma with a "phantom charge" corresponding to the factor $\ell n \prod_i (z_i - z_0)$. Due to perfect screening of the plasma as $r \to \infty$, the quasi particle charge turns out to be $q/e = \frac{1}{m}$. In an analogous argument, since the factor $\prod_i (z_i - z_0)$ adds one unit of angular to each particle, the fluid is displaced radially leaning the $\ell = 0$ state vacant. Since the density of electrons is $\frac{1}{m}$ per ℓ state, it follows that $q/e = \frac{1}{m}$. Similar arguments have been given by Haldane[3] and by Halperin.[4]

Another approach to determining the quasi particle charge is through the use of the adiabatic theorem. Arovas, Wilczek and the author[6] calculated the change of phase γ of the many body state $\Psi(t)$ as z_0 adiabatically moves around a circle of radius R enclosing a flux Φ . From gauge invariance, the phase a particle of charge q would acquire in such a process is

$$\frac{q}{\hbar c} \oint \vec{A} \cdot d\vec{\ell} \quad = -2\pi \left(\frac{q}{e}\right) \phi / \phi_0 \ . \tag{4}$$

Therefore, one finds $q/e = -\gamma\phi_0/2\pi\phi$. To determine γ , we note that if a Hamiltonian $H(z_0)$ depends on a parameter z_0 which slowly traverses a loop, then in addition to the usual phase $\int^t E(t')dt'/\hbar$, where $E(t')$ is the adiabatic energy, there is an extra contribution $\gamma(t)$ to the phase of $\Psi(t)$ regardless of how slowly the path is traversed. $\gamma(t)$ satisfies[7,8,9,10]

$$\frac{d\gamma(t)}{dt} = i\left\langle \Psi(t) \middle| \frac{d\Psi(t)}{dt} \right\rangle \Big/ \langle \Psi(t) | \Psi(t) \rangle \ . \tag{5}$$

From Eqn. 2 we have

$$\frac{d\psi_m^{+z_0}}{dt} = \sum_i \ln[z_i - z_0(t)] \psi_m^{+z_0} \tag{6}$$

so that

$$\frac{d\gamma}{dt} = i \left\langle \psi_m^{+z_0} \middle| \frac{d}{dt} \sum_j \ln(z_j - z_0) \middle| \psi_m^{+z_0} \right\rangle \ . \tag{7}$$

Since the one-electron density in the presence of the quasi hole is given by

$$\rho^{+z_0}(z) = \left\langle \psi_m^{+z_0} \middle| \sum_j \delta(z_j - z) \middle| \psi_m^{+z_0} \right\rangle \tag{8}$$

we have

$$\frac{d\gamma}{dt} = i \int dxdy \ \rho^{+z_0}(z) \frac{d}{dt} \ln[z - z_0(t)] \ . \tag{9}$$

Since we are interested in γ for loop radius $R >> a_0$, we write $\rho^{+z_0}(z) = \rho_0 + \delta\rho^{+z_0}(Z)$, where ρ_0 is the electron density in the ground state. For the ρ_0 term, if z_0 is integrated clockwise around a circle of radius R, values of $|z| < R$ contribute $2\pi i$ to the integral while $|z| > R$ contribute zero. Thus, the ρ_0 contribution to γ is

$$\gamma_0 = i \int_< dxdy\ \rho_0\ 2\pi i = -2\pi\langle n\rangle_R = -2\pi\nu\phi/\phi_0\ , \tag{10}$$

where $\langle n\rangle_R$ denotes the expected number of electrons in the loop. Combining Eqs. 4 and 10 we obtain $q/e = \nu$, in agreement with Laughlin's approach. A similar calculation gives $q/e = -\nu$ for a quasi particle.

<u>FRACTIONAL STATISTICS</u>

To determine the statistics satisfied by the quasi particles we consider the two quasi particle state

$$\psi_m^{z_a z_b} = \prod_i (z_i - z_a)(z_i - z_b)\psi_m \cdots \tag{11}$$

As in the calculation of q, we carry z_a around a circle of radius R. If $|z_b|$ is outside the circle by a distance large compared to the size of a_0 of the quasi particle charge form factor, the above analysis for γ is unchanged. However if z_b is within the circle with $|z_b| - R << -a_0$, the change of $\langle n\rangle_R$ due to the presence of the quasi particle z_b is $-\nu$. Therefore, there is an extra phase $\Delta\gamma = 2\pi\nu$ arising from z_a encircling z_b, which is naturally ascribed to a "statistical phase." For $\nu = 1$, where $q/e = 1$, $\Delta\gamma = 2\pi$ and a phase change $\Delta\gamma/2 = \pi$ occurs when two quasi particles are interchanged, i.e. $e^{i\Delta\gamma} = -1$ which is the correct result for fermions. For general ν , the phase factor arising from quasi particle interchange is $e^{i\pi\nu} = (-1)^\nu$. Particles having fractional statistics have been termed anyons[9], i.e. any statistics. In general, the statistical phase factor for a many quasi particle state is

$$f = \exp i\nu \sum_{a<b} \theta_{ab} \tag{12}$$

where θ_{ab} is the angle between z_a and z_b. A new feature relative to Bose of Fermi statistics is that the change of phase on interchanging a and b depends on how many other quasi particles are encircled in taking a given path from the initial to find configuration. This dependence on the positions of other particles exactly cancels in the Bose or Fermi case since $e^{i2\pi\nu} = 1$ for ν integer.

DISCUSSION

We have seen that similar arguments describe the origin of fractionally charged excitations in commensurate quasi 1D systems and in the fractional quantum Hall effect. In both cases the incompressibility of the electron liquid ensures that q/e is fractionally quantized.

The full consequences of the statistics have yet to be worked out. Recently, Arovas, Wilczek, Zee and the author have solved for the free energy of a dilute anyon gas, i.e. the second virial coefficient $B_2(T)$. One finds that B_2 exhibits periodic behavior as a function of the statistical phase $2\pi\nu$, smoothly changing from Bose to Fermi to Bose, but with a cusp at Bose statistics. It is clear that statistical effects play a significant role in determining the properties of a collection of quasi particles.

ACKNOWLEDGEMENTS

This work was supported in part by the national Science Foundation under grants DMR82-16285 and PHY77-27084, supplemented by funds from the National Aeronautics and Space Administration.

REFERENCES

1. W. P. Su, A. J. Heeger and J. R. Schrieffer, Phys. Rev., B22, 2099 (1980).
2. R. B. Laughlin, Phys. Rev. Lett., 50, 1395 (1983).
3. F. D. M. Haldane, Phys. Rev. Lett., 51, 605 (1983).
4. B. I. Halperin, Phys. Rev. Lett., 52, 1583 (1981).
5. W. P. Su and J. R. Schrieffer, Phys. Rev. Lett., 46, 738 (1981).

6. D. Arovas, J. R. Schrieffer, and F. Wilczek, Phys. Rev. Lett., 53, 722 (1984).

7. M. V. Berry, Proc. Ray. Soc. London Ser., A392, 45 (1984).

8. B. Simon, Phys. Rev. Lett., 51, 2167 (1983).

9. F. Wilczek and A. Zee, Phys. Rev. Lett., 52, 2111 (1984).

10. L. Schiff, Quantum Mechanics (McGraw-Hill, New York 1955) p. 290.

11. D. Arovas, J. R. Schrieffer, F. Wilczek and A. Zee, to be published.